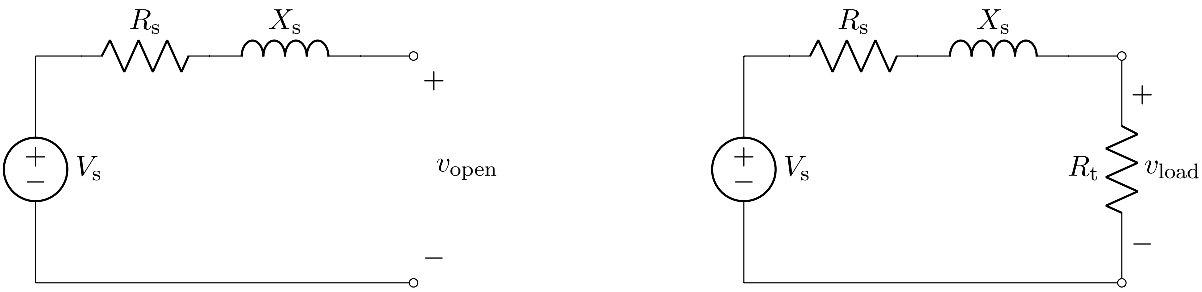
Source-impedance estimation from voltage magnitude with/without resistive load

Purpose: show that this method can give a wrong estimation of short-circuit current if the source impedance has a substantially different angle from the test-load impedance.
NT, 2020-06.

A simple model of the situation is shown by the following two circuits. The system seen from a two-terminal measurement point is treated as a Thevenin source with fixed voltage amplitude V_{s} and impedance $R_{\mathrm{s}} + \mathrm{j}X_{\mathrm{s}}$.



The impedance is estimated by comparing two situations: the open-circuit voltage v_{open} (left), and the voltage v_{load} with a test-load R_{t} connected (right).

The main purpose of impedance estimation in electrical installation work is to assess the *current magnitude* in a short-circuit such as L-L, L-N or L-E. The L-E case gets particular attention for checking earth-fault disconnection times.

The actual short-circuit current magnitude at the terminals in the above circuit is

$$i_{\mathrm{sc}} = \left| \frac{V_{\mathrm{s}}}{R_{\mathrm{s}} + \mathrm{j}X_{\mathrm{s}}} \right| = \frac{V_{\mathrm{s}}}{\sqrt{R_{\mathrm{s}}^2 + X_{\mathrm{s}}^2}},$$

regardless of whether R_{t} is included in the circuit.

The next page shows how a phasor-based and magnitude-based method estimate the source impedance by comparing v_{open} and v_{load} and knowing R_{t}.

In practice there are plenty of further complications. For example, the voltage at the measurement point can also change during a measurement, due to nearby loads that turn on or off, or to tap-changer operation. Non-sinusoidal waveforms may affect the result. And the local voltage measurements made at different times must be measured as phasors with a consistent relation to the the remote source: there are various options, with risk of disturbance when things change in the system during the measurement.

These are matters that can be improved to some extent by careful and repeated measurement — they are not fundamental limitations of the principle. The important point is that the magnitude-only principle has a fundamental limitation that the phasor-based one doesn't.

Manufacturers of testers will doubtless be aware of these limitations if using a simple method of switched resistance and voltage magnitudes. Some might be using more sophisticated methods even for their higher-current (non-RCD) tests. This derivation was just intended to illustrate the theoretical point that even the impedance magnitude and short-circuit magnitude can be estimated very wrongly if the source is significantly reactive.

Measurement of the voltages as phasors

If the measured voltages v_{open} and v_{load} can be measured as phasors, with angles relative to the source voltage, the impedance can be properly estimated for all conditions.

Take the source voltage as the reference angle, so $\bar{V}_{\text{s}} = V_{\text{s}}\underline{/0}$, where the bar in '$\bar{V}_{\text{s}}$' indicates a phasor rather than just the magnitude 'V_{s}'.

In this case, the measured voltages in the two circuits are

$$\bar{v}_{\text{open}} = \bar{V}_{\text{s}}, \quad \text{and} \quad \bar{v}_{\text{load}} = \frac{R_{\text{t}}}{R_{\text{t}} + R_{\text{s}} + \mathrm{j}X_{\text{s}}}\,\bar{V}_{\text{s}}, \qquad \text{so} \qquad \bar{v}_{\text{load}} = \frac{R_{\text{t}}}{R_{\text{t}} + R_{\text{s}} + \mathrm{j}X_{\text{s}}}\,\bar{v}_{\text{open}}.$$

Rearrangement gives the source's *complex* impedance, $R_{\text{s}} + \mathrm{j}X_{\text{s}}$:

$$\frac{\bar{v}_{\text{open}}}{\bar{v}_{\text{load}}} = \frac{R_{\text{t}} + R_{\text{s}} + \mathrm{j}X_{\text{s}}}{R_{\text{t}}} = 1 + \frac{R_{\text{s}} + \mathrm{j}X_{\text{s}}}{R_{\text{t}}}, \qquad \text{from which} \qquad R_{\text{s}} + \mathrm{j}X_{\text{s}} = \frac{\bar{v}_{\text{open}} - \bar{v}_{\text{load}}}{\bar{v}_{\text{load}}}\,R_{\text{t}}.$$

This impedance is expected to be the genuine value, as it's simply working backwards from the circuit solutions. The correct short-circuit current magnitude i_{sc} would therefore be calculated if using this method, subject to the pratical limitations of measurement mentioned earlier. (Try it with some numbers if in doubt.)

Measurement of the voltages as magnitudes alone

In the simple type of estimation that measures just the voltage amplitudes, the voltages can be used in a similar equation to the above, but using only the magnitudes of the voltages and impedance.

Define the *estimated* magnitude of the source impedance as Z'_{s}, calculated from

$$Z'_{\text{s}} = \frac{v_{\text{open}} - v_{\text{load}}}{v_{\text{load}}}\,R_{\text{t}}.$$

Putting in the magnitudes of the measured voltages, in terms of the source voltage magnitude, this becomes

$$Z'_{\text{s}} \;=\; \frac{V_{\text{s}} - \frac{V_{\text{s}}R_{\text{t}}}{\sqrt{(R_{\text{t}}+R_{\text{s}})^2+X_{\text{s}}^2}}}{\frac{V_{\text{s}}R_{\text{t}}}{\sqrt{(R_{\text{t}}+R_{\text{s}})^2+X_{\text{s}}^2}}}\,R_{\text{t}} \;=\; \left(\frac{\sqrt{(R_{\text{t}} + R_{\text{s}})^2 + X_{\text{s}}^2}}{R_{\text{t}}} - 1\right) R_{\text{t}}$$

or looking a little nicer,

$$Z'_{\text{s}} \;=\; \sqrt{(R_{\text{t}} + R_{\text{s}})^2 + X_{\text{s}}^2} \;-\; R_{\text{t}}.$$

Compare the short-circuit current estimate i'_{sc} based on the estimated Z'_{s}, with the actual short-circuit current i_{sc} that was shown earlier:

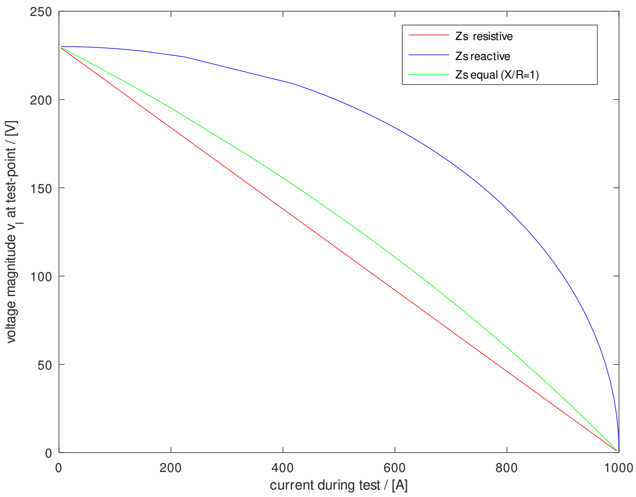
$$i'_{\text{sc}} \;=\; \frac{V_{\text{s}}}{\sqrt{(R_{\text{t}} + R_{\text{s}})^2 + X_{\text{s}}^2} \;-\; R_{\text{t}}} \qquad \text{whereas actually} \qquad i_{\text{sc}} \;=\; \frac{V_{\text{s}}}{\sqrt{R_{\text{s}}^2 + X_{\text{s}}^2}}.$$

Notice from the above that:

- If $R_\mathrm{t} = 0$, then $i'_\mathrm{sc} = i_\mathrm{sc}$.
  In words, "if you test with a short-circuit, you find the short-circuit current!".
  However, a fuse might go before you even get anywhere near a full period. . . .
- If $X_\mathrm{s} = 0$, then also $i'_\mathrm{sc} = i_\mathrm{sc}$.
  The more general point that this comes from is that if the test load has the same impedance angle (X/R ratio) as the source, then the estimate will match the expected value. Our calculations assumed just a resistive test load, which only works perfectly with a resistive source.
- If $X_\mathrm{s} \gg R_\mathrm{s}$ and $R_\mathrm{t} \gg X_\mathrm{s}$, then $i'_\mathrm{sc} \gg i_\mathrm{sc}$.
  This means that when reactance is dominant in the source impedance and the test load takes a small current compared to the short-circuit currents, the short-circuit current will be estimated to be more (potentially by many times) than it really is.

If the purpose of a measurement is to check whether a short-circuit rating is exceeded, this type of error where $i'_\mathrm{sc} > i_\mathrm{sc}$ is on the side of caution. If the purpose is to ensure disconnection times, for which higher current more easily fulfils the requirement, the error could give false confidence. However, disconnection times are (I suspect) more likely to need to be checked towards the weaker outlying parts of the system, where the smaller cables make it likely that resistance would dominate the source impedance. Furthermore, the system design should involve calculations based on the components installed, not just reliance on installation testers. The testers would be useful for verifying that the situation isn't worse than expected.

The plot on the right is a way of visualising how the different estimates arise. The curves are for three different angles of source impedance, from purely resistive to purely reactive. All have the same magnitude, giving 1 kA short-circuit current. The varied test-current is obtained by different values of test resistor, from very low up to 60 Ω.



The magnitude-based method of estimating source-impedance with a relatively high test impedance can be seen as finding the gradient at the low-current end, and extrapolating it to find the short-circuit current.

At low test-currents the test-resistance is the main part of the total impedance, so the current is largely in phase with the source voltage. In this situation, the voltage drop in a reactive source impedance will add largely in quadrature with the source voltage, giving a much shallower gradient than would happen with the same amount of resistive source impedance. This shallow gradient is what leads to an excessive estimate of short-circuit current, as it ignores the fact that the gradient later goes down more steeply.

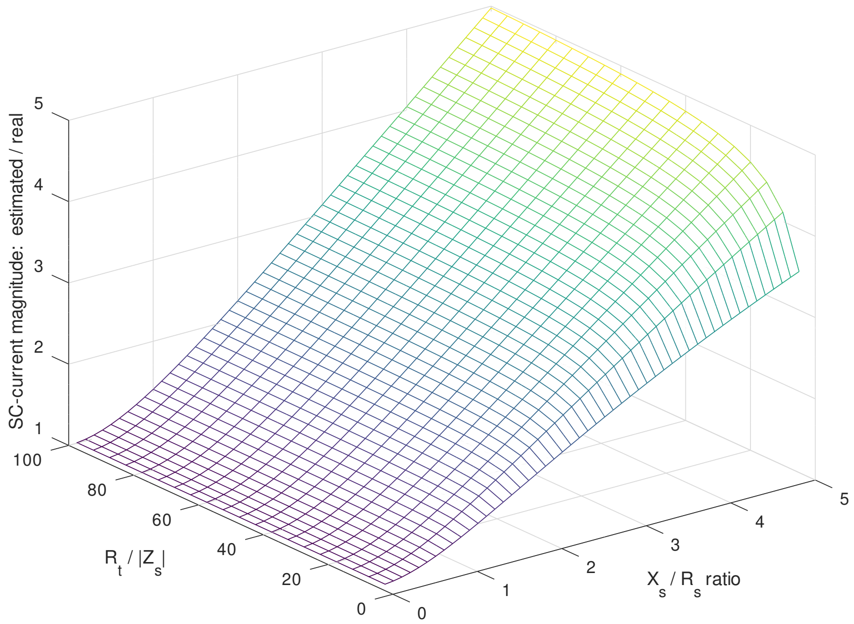
Variation of error factor with X/R and R_t

The following plot shows the ratio of estimated to actual short-circuit current $\frac{i'_{sc}}{i_{sc}}$ when using magnitude-based estimation. It is plotted as a function of the source's X_s/R_s ratio and of the ratio of test-resistance to source impedance magnitude. Ideally the plotted surface would lie flat across the bottom, where $\frac{i'_{sc}}{i_{sc}} = 1$.

If the short-circuit current at some point is as low as 230 A, then a tester that uses a load of 10 A has a $R_t/|Z_s|$ of 23. This ratio increases for stronger points in the system. The points where the surface curves downwards as $R_t/|Z_s|$ moves towards zero are therefore not realistic for installation testing.

For the plausible values of $R_t/|Z_s|$, the estimated short-circuit current exceeds the actual when X_s/R_s rises beyond being close to zero. At the ratio that would be found on the output of a strong transformer, the estimate could be several times too much.

In a domestic or similar situation with small conductors it is reasonable to assume $X_s/R_s < 0.1$, which is why the magnitude method with a resistive load tends to work acceptably. In a few cases where short-circuit current is very high at a meter position and the X/R ratio is moving towards 1, there will be no issue with disconnection times, so an overestimated current would not be unsafe.



Generated by running the following in GNU Octave.

```
XR = [ 0.1:0.1:5 ]';    % X/R ratios
RtZ = [ 5:5:100 ];      % ratio of test to source impedances
Zm = 1;   % magnitude of source impedance (use 1 for easy comparison)
Rt = ones(length(XR),1) * (Zm*RtZ);
R = (Zm./sqrt(1+XR.^2)) * ones(1,length(RtZ));
X = XR.*R;
iest = 1./( sqrt((Rt+R).^2+X.^2) - Rt );
mesh( XR, RtZ, iest.' );
xlabel('X_s / R_s ratio')
ylabel('R_t / |Z_s|')
zlabel('SC-current magnitude:  estimated / real');
```